A viscoelastic model of rough-wall boundary-layer noise

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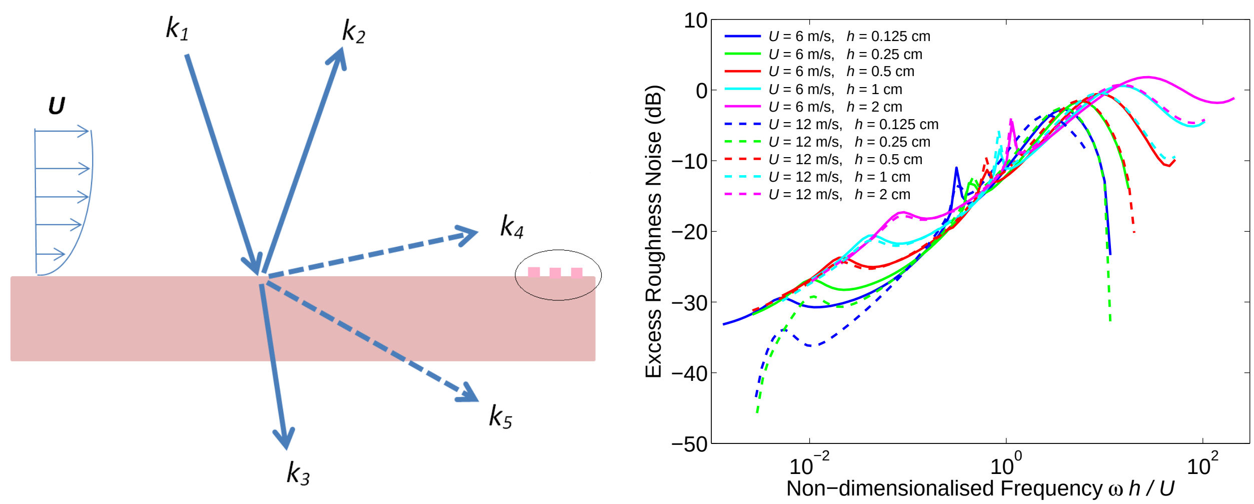
We present an analytical framework that can be used for quantitative estimation of the effect of of wall roughness on turbulent boundary-layer noise. For this purpose we extend the viscoelastic analogy for boundary-layer noise (the well-known analogy between the motion of a viscoelastic medium and a viscous fluid) that was proposed in our previous work [1]. We compare our results with other well-known models and experimental data [2, 3, 4].

Our analysis is restricted to that of a slightly compressible fluid, i.e. flows for which Mach number $M = U/c_l$ is much less than unity, where c_l is the speed of sound and U is the velocity of unperturbed flow, far from an underlying surface. The surface is assumed elastic with the speed of elastic waves in its material much greater than U.



(a) (b)

Figure 1: (a) Transformation of a vorticity perturbation (shear wave) with wavevector k_1 in a turbulent boundary-layer into longitudinal (sound) waves with wavevectors k_4 and k_5 (and a shear component k_2) at an elastic wall. (b) The increase in radiated flow noise due to roughness, relative to that from a smooth boundary, for a 20 mm thick air-backed plate in water as a function of dimensionless 'roughness' parameter $\omega h/U$ assuming $L = 100$ m.

In this approach the turbulent flow is modeled as a system of random shear (vortex) waves propagating in a 'soft' (rubber-like) medium, as in Fig. 1a. It can be shown [1] that the properties of the medium can be mapped to the parameters of turbulent flow in the boundary layer with the condition $c_s = U - i\nu k_s$, where c_s is phase velocity of the shear waves, k_s is their wave number, and ν is is the kinematic viscosity of the fluid. This equation can also be expressed in terms of the effective shear modulus of the medium $\mu = \mu_r + i\mu_i$, with $c_s = \sqrt{\mu/\rho}$, so that $\mu_r \simeq \rho U^2$ and $\mu_i \simeq 2\rho\omega\nu$, where ρ is fluid density and $\omega = k_s U$ is frequency of shear wave.

Conventional methods of elastic wave transformation can then be applied to study the process of flow noise generation near the elastic boundary by considering the transformation of vorticity perturbations (shear waves) into sound (longitudinal) waves at the boundary.

The mean velocity profile in a turbulent boundary layer is described by the well-known function [5]

$$u(z)/v_\tau = F(\zeta), \tag{1}$$

where $\zeta = v_\tau z/\nu$, z is distance from the wall, $F(\zeta) = (1/\kappa)\ln(\zeta) + B$, $\kappa = 0.41, B = 8.5$, $v_\tau = U\sqrt{c_f/2}$ is frictional velocity, and c_f is the friction drag coefficient which is generally modified by roughness. For the turbulent boundary layer over a rough wall Eq. (1) remains approximately the same, but with the substitution $\zeta = z/h$, where h is the effective roughness height [5]. It can be seen that, as the first approximation, this change can be captured by introducing the effective viscosity

$$\nu_e = \nu(1 + v_\tau h/\nu) = \nu\left(1 + \sqrt{c_f/2}(h/L)\mathrm{Re}_L\right), \tag{2}$$

so that $\nu_e = \nu$ for at the limit of smooth wall ($h = 0$). Parameter $\mathrm{Re}_L = UL/\nu$ is the Reynolds number of the turbulent flow, L is the downstream dimension, and ratio h/L is the relative roughness. For the 'roughness control' turbulent boundary layer ($v_\tau h/\nu > 5$) the following formula can be used [4]

$$c_f = [2.635 + 0.618\log(L/h)]^{-2.57}. \tag{3}$$

With the effective viscosity of Eq. (2) we can define the effective shear modulus for the 'soft medium, $\mu_r \simeq \rho U^2$, $\mu_i \simeq 2\rho\omega\nu_e$, and then apply the theoretical framework proposed for a smooth surface [1] to estimate the dependency of roughness-induced turbulent boundary-layer noise on the parameter h/L and elastic properties of the wall.

We have calculated turbulent boundary layer noise over a rough wall for a range of parameters, including Reynolds number of the flow, relative roughness and materials properties of the underlying wall. An illustrative example of these result is presented in Fig. 1b.

References

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