Chase vs. Corcos TBL Loading

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Two of the more frequently used models for the turbulent boundary layer (TBL) pressure excitation of structures are by Corcos [1] and Chase [2]. Analytically both are based on the divergence of the Navier-Stokes equation for steady, incompressible flow, which in Cartesian indicial notation is

$$\partial^2 p/\partial x_i^2 = -\,2\rho(\partial U_i/\partial x_j)\,(\partial u_j/\partial x_i) - \rho_o\,\partial^2[u_i\,u_j - \langle u_i\,u_j\rangle_t]\,/\,\partial x_i\,\partial x_j \tag{1}$$

where p is the fluctuating pressure, ρ is the ambient fluid density, U is the mean velocity, and u is the fluctuating velocity. The brackets $\langle\cdots\rangle_t$ indicate the mean value over time, t. Solving this equation is challenging because the velocity products make it a highly non-linear problem analytically and they are difficult to measure.

Corcos used measurements of the spatial correlation function of the surface pressure, *i.e.* in the (x_1, 0, x_3) plane,

$$\mathrm{R}_{pp}(\xi,\ \eta,\ \tau) = \langle\, p(x_1, 0, x_3, t)\ \ p(x_1+\xi, 0, x_3+\eta, t+\tau)\,\rangle_t \tag{2}$$

and its frequency transform, $\Gamma_{pp}(\xi,\ \eta,\ \omega)$, to conclude there is a dominate similarity with the non-dimensional variables, $\omega\xi/U_c$ and $\omega\eta/U_c$., where U_c is the convection velocity. In particular, using correlation measurements in the stream-wise direction (x_1) and in the cross-wise direction (x_3), Corcos determined a similarity relationship

$$\Gamma_{pp}(\xi,\ \eta,\ \omega) = \Phi_{pp}(\omega)\ \mathrm{A}(\omega\xi/U_c)\ \mathrm{B}(\omega\eta/U_c)\ \exp(i\omega\xi/U_c) \tag{3}$$

where $\Phi_{pp}(\omega)$ is the single point frequency spectrum of the surface pressure. This is an attractive formulation because it is separable in the space coordinates making it amenable to analytical representations of the coupling to structural vibration modes (at least for the separable modes of a simply supported, rectangular plate). Also, if A and B are given as decaying exponentials, $\mathrm{A} = \exp(-\alpha\,\omega|\xi|/U_c)$, $\mathrm{B} = \exp(-\beta\,\omega|\eta|/U_c)$, (which agree well with measured data), the model represents a decaying wave propagating in the x_1 direction. Transforming to the wavenumber domain (k_1, k_3) gives a wavenumber-frequency spectrum of the form

$$\Phi_{pp}(k_1, k_3, \omega) = \Phi_{pp}(\omega)\ [\alpha\,\beta/\pi^2 k_c^2]\ /\ [\alpha^2 + (k_1/k_c - 1)^2]\ [\beta^2 + (k_3/k_c)^2\ ] \tag{4}$$

where $k_c = \omega/U_c$. α and β are empirically determined constants

Chase, on the other hand, used measurements and models of the TBL velocities to determine a representation for the surface pressure using the Fourier transformed and integrated Eq. 1,

$$\Phi_{pp}(k_1, k_3, \omega) = \rho^2 (k_1^2+k_3^2) \int_0^\infty dx_2 \int_0^\infty dx'_2 \exp[-(k_1^2+k_3^2)(x_2+x'_2)] [S_M(k_1, \zeta, k_3, \omega) + S_T(k_1, \zeta, k_3, \omega)] \quad (5)$$

where S_M and S_T are the cross-spectra of the velocity product terms in Eq. 1 at a spacing $\zeta = x_2 - x'_2$ above the surface (and the assumption is made that $dp/dx_2|_{x_2=0} = 0$). Measurements of the spatial correlation in the fluctuating stream-wise velocity, $\Gamma_{u_1u_1}(\xi, \zeta, \eta, \omega)$, indicate there is a general similarity form in the wavenumber domain given by

$$S_{u_1u_1}(k_1, \zeta, k_3, \omega) = F[(k_1^2+k_3^2)^{1/2} x_2] \Phi_0(k_1, k_3, \omega) \quad (6)$$

Using other assumed functional shapes and asymptotic evaluations of integrals, Chase obtained the following results for his basic model

$$\Phi_{pp}(k_1, k_3, \omega) = C \rho^2 U^3 (k_1^2+ak_3^2) / [k_1^2 + k_3^2 + \mu(k_1 - k_c)^2 + (1/b\delta)^2]^{5/2} \quad (7)$$

where δ is the TBL thickness and C, a, b, and μ are empirically derived constants.

Figure 1 compares the Corcos and Chase basic models with measured data in the low wavenumber region for $k_3 = 0$. The Corcos model over predicts and the basic Chase model under predicts the data.

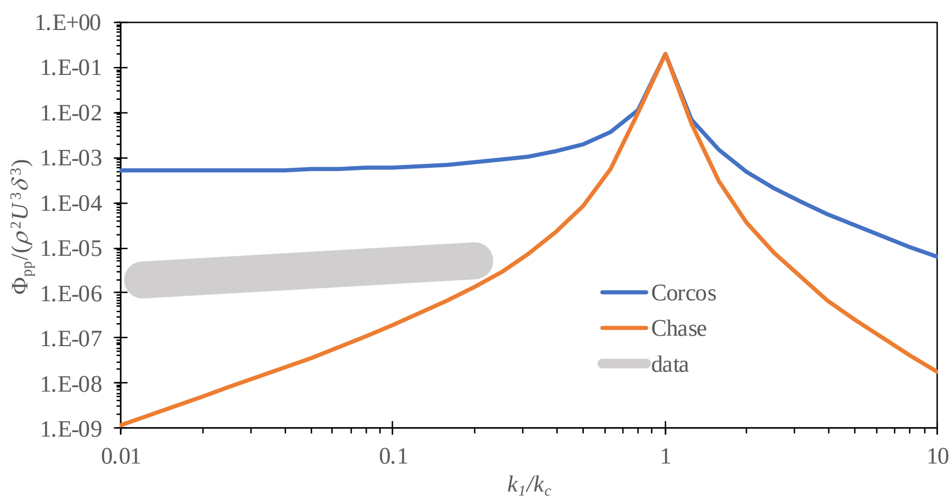

Figure 1: Comparison of TBL surface pressure wavenumber-frequency spectra

There have been many attempts to modify these models to agree better with the measured data. This paper discusses some of these modifications and presents a suggested revision to Chase's derivations to improve that model. Also discussed are the coupling of the TBL pressures to structural vibrations, recent direct numerical solutions (DNS) to the Navier-Stokes equations, and areas were further research is needed.

References

[1] G.M. Corcos, "The structure of the turbulent pressure field in boundary-layer flows," *Journal of Fluid Mechanics*, vol. 18, no. 3, pp. 353–375, 1964.
[2] D. M. Chase, "Modeling the wavevector-frequency spectrum of turbulent boundary layer wall pressure," *Journal of Sound and Vibration*, vol. 70, pp. 29-67, 1980.